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COMPARATIVE ANALYSIS OF NATURAL FREQUENCY OF TRANSVERSE VIBRATION OF A CANTILEVER BEAM BY ANALYTICAL AND EXPERIMENTAL METHODS

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Abstract

Vibration analysis of any mechanical systems is very important since excess vibrations leads to failure of the mechanical systems. There are many methods and instruments available in the engineering field for measuring the mechanical vibrations in mechanical systems. In this present work transverse vibrations of a cantilever beam are analyzed analytically and experimentally. In our present work FFT analyzer is used to measure the natural frequency of vibration of a cantilever beam. Initially the natural frequency of the cantilever beam is calculated analytically and then natural frequency was found out by experimentally by using FFT and the results were compared. Both analytical and experimental results were found to be almost the same. The present work shows a standard procedure for the determination of natural frequency of vibration of a cantilever beam by analytical and experimental methods by using FFT analyzer.

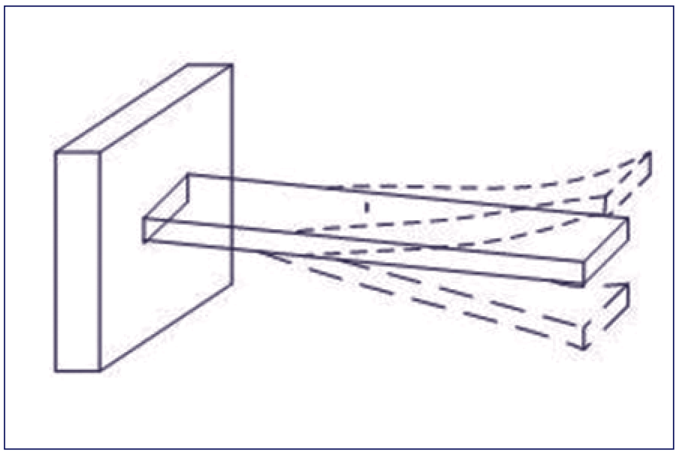
***Key Words:* Cantilever beam, Natural frequency**

1. INTRODUCTION

A straight, horizontal cantilever beam is deformed into curve when it is subjected to a vertical load. After the removal of Load, the beam regains its original shape as it was prior before subjected to load, but due to inertia the beam stays in motion. Which results in vibration of beam at characteristic frequency [6]. In this paper a cantilever beam with rectangular cross section for transverse vibration is investigated. Device used are Accelerometer and Fast Fourier Transform Analyzer. Most extensively studied among structural elements are beam as it finds many engineering applications. Structure like Long span bridges, robot arm and tall buildings are modeled with beam like element, beams as well as the presence of cracks affects the dynamic response of the structure significantly. It can result into failure of structure in a very extreme way. To predict the failure, vibration monitoring can be used to detect changes in the dynamic responses and/or dynamic characteristics of the structure [7]. One of the most important tasks in designing of mechanical system is Vibration Analysis. Design engineer has to take into consideration information like dynamic behaviour of structure of machine tool, effect of vibration absorber on rotating machineries. This information helps to design system to control the excessive amplitude of the vibration [2]. Before the advent of computers, seeking the analytical solution was one of the most popular ways to solve an engineering problem. In comparison with numerical methods, the analytical solutions have the advantage of elegance and are time-saving. However, for most practical engineering problems no analytical solutions can be found, unless they are significantly simplified. Because of this drawback of the analytical method, it has gradually been replaced by numerical methods after the invention of computer, particularly in recent years. One of the greatest shortcomings of the numerical approach is that it is time-consuming, but because of its powerful ability for solving the practical problems this drawback seems allowable [1]. An intelligent FFT analyser is capable of adapting its operating parameters on the basis of the signal spectrum set up and characters. The realised instrument is equipped with auto configuration capability. The experimental tests carried out on a large number of signals highlight the instrument capability of correctly detecting a good frequency resolution for any signal spectrum type [3]. In this paper a straight horizontal cantilever beam is subjected to free vibration due to which it will vibrate at its natural frequency. The aim is to study the vibration of thin film cantilever beam. The frequency equations are solved to show the output frequencies and the mode shapes related to each frequency [4]

2. THE CANTILEVER BEAM

Figure No. 1: The beam under free vibration



A Cantilever beam is a beam whose one end is fixed and another end is free. When a beam is subjected to load it withstands the load by resisting against bending. Bending moment is the reaction of the beam, when a beam is subjected to bending force due to its own weight, external load, etc. When a straight horizontal cantilever beam is exposed to vertical load, it will bend downwards and after removal of load will regain its original shape but will vibrate at natural frequency due to inertia. Most fundamental structure of any structure from decades is cantilever beam. The cantilever beam is much greater in length as compared to width and depth. In addition, cantilever beams may be straight or curved, with rectangular or circular cross sections. Figure 1 shows a cantilever beam with a rectangular cross section. The design and shape of cantilever beam is application specific, its weight, size, shape all parameters changes based on application. For example, one of the most common applications of a cantilever beam can be the "fixed wing" in meters is designed as a beam for some preliminary analysis to help lift the plane and make it fly. In Micro Electrical Mechanical Systems (MEMS), micro cantilever beams are used in radio frequency filters and resonator [4].

3. NATURAL FREQUENCY FROM ANALYTICAL CALCULATION

Beam's Specifications: L = 0.3 m, b = 0.019 m, t = 0.002 m

M. I. of cantilever beam (I) = $(1/12)xbt^3$

$= (1/12)x0.019 \times (0.002)^3 = 1.267 \times 10^{-11}\ m^4$

Area of cross section of cantilever beam (A)

$= b \times t = 0.019 \times 0.002 = 3.8 \times 10^{-5}\ m^2$

E = 210*GPa* (210x10^9 *N/m^2*) ---For steel.

$\rho = 7800\ kg/m^3$ ----For steel.

For 1st mode

$$\omega = 1.875^2\sqrt{\frac{EI}{\rho A L^4}} = 1.875^2\sqrt{\frac{210 * 10^9 * 1.267 * 10^{-11}}{7800 * 3.8 * 10^{-5} * 0.3^4}}$$

= 117.03rad/sec, & $f_n = \frac{\omega}{2*\pi}$= 18.62Hz

For 2nd mode

$$\omega = 4.694^2\sqrt{\frac{EI}{\rho A L^4}} = 4.694^2\sqrt{\frac{210 * 10^9 * 1.267 * 10^{-11}}{7800 * 3.8 * 10^{-5} * 0.3^4}}$$

=733.5rad/sec, & $f_n = \frac{\omega}{2*\pi}$= 116.74 Hz

For 3rd mode

$$\omega = 7.855^2\sqrt{\frac{EI}{\rho A L^4}} = 7.855^2\sqrt{\frac{210 * 10^9 * 1.267 * 10^{-11}}{7800 * 3.8 * 10^{-5} * 0.3^4}}$$

=2054.03rad/sec, & $f_n = \frac{\omega}{2*\pi}$=326.9Hz

4. THE FFT ANALYZER

In FFT spectrum analyzer, the input signal is digitized and that to at high sampling rate which is similar to oscilloscope. The digital time record obtained as a result is then transformed using a algorithm known as Fast Fourier transform algorithm, into a Frequency Spectrum. The function of the signal analyzer is to measure the magnitude of an input signal compared frequency with the maximum frequency range of instrument. The amplitude of vibration is recorded on the basis of its evolution versus the frequency at that the signal appears. The original time domain signal is converted into Frequency domain view. Single Channel FFT and Multichannel type is specially used for determining Single vibration parameter from one or more sources.

4.1. Benefits of FFT analyzer:

4.1.1. Quick capture of waveform: In view of the very fact that the waveform is analyzed digitally, the waveform can be captured in a comparatively very short time, and then the afterwards analyzed. This short capture time will have many advantages – it will afford the capture of transients or short-lived waveforms.

4.1.2. Able to capture non-repetitive events: The short capture time implies that the FFT analyzer will capture non-repetitive waveforms, giving them a capability, which will be not possible with other spectrum analyzers.

4.1.3. Able to analyze signal phase: As part of the signal capture method, information is gained which might be processed to reveal the phase of signals.

4.1.4. Waveforms can be stored using FFT technology: It will be possible to capture the waveform and analyze it later and this should this be required.

4.2. Disadvantages of the FFT analyzer:

4.2.1. Frequency limitations: The important limitation of the frequency and bandwidth of FFT spectrum analyzers is that the analogue to digital converter, ADC that is used to convert the analogue signal into a digital format. Whereas technology is raising this component is still places a serious limitation on the higher frequency limits or the bandwidth if a down-conversion stage is employed.

4.2.2. Cost: The high level of performance needed by the ADC implies that, this item could be a terribly very high-cost item. Additionally, to all the other processing and display circuitry required, this leads to the prices rising for this stuff.

5. EQUIPMENT USED FOR EXPERIMENTATION

Apparatus used to perform the real experiment are Impact Hammer, Accelerometer, Multi-channel Vibration Analyzer (DEWESoft-DEWE-43), A computer or a laptop loaded with software for modal analysis. Test-specimen that is a cantilever beam which is held in a fixture, power supply to be given to the computer and the vibration analyzer, connecting cables for the impact hammer and the accelerometer, fasteners and spanner to assemble rigidly the specimen in the fixture, adhesive wax to fix the accelerometer.

5.1. Impact Hammer: This seems like an ordinary hammer however its head is fitted with a load cell and contains electronic circuitry and an output cable that will be connected to vibration analyzer. On hitting the impact hammer on any structure an impulsive force is applied to the structure which is a cantilever beam in our present work. An equal and opposite force is detected by the load cell fitted within the head of the hammer. This generates an electrical signal that is given to vibration analyzer that analyzes the signal, compares with the signal received from accelerometer connected to the structure, and this information is used to develop FRF (Frequency Response Function) and finally the natural frequencies of the structure are found out.

5.2. Accelerometer: An accelerometer is a device (a transducer) which when attached to a vibrating structure gives out the electric signals which are proportional to the acceleration. This signal is given to a vibration analyzer that processes and analyzes the signal. Just in case of our experiment, the signal coming from accelerometer fitted to the cantilever is analyzed with respect to the one received from the impact hammer in order to search out the natural frequencies of the cantilever.

5.3. Vibration Analyzer: Vibration Analyzer is an electronic equipment that processes and analyzes the signals received from transducers used in vibration measurement such as impact hammer, accelerometer, digital tachometer and many more. It has various channels that is it can receive number of electric signals at same time. The vibration analyzer has terribly refined electronic circuits and works along with with a computer. Fast Fourier Transform (FFT) is an algorithm often used for analysis of the electric signals which gives frequency components and their corresponding amplitudes present in the signals.

5.4. A Computer or a Laptop loaded with software for modal analysis: Additionally, to the software used together with the vibration analyzer, the software are available now-a-days devoted exclusively to the modal analysis. Such software helps inputting data to the computer related to geometry of the structure, location of placing of the accelerometer on the structure as well as points of hitting of the impact hammer and their directions and so on. Also, there are facilities to choose different type of analysis required, ranges of various parameters of interest. Such software essentially processes the signals received from the impact hammer and accelerometer, carrying out their FFTs, finding FRF, getting Mode Indicator Function, carrying out curve fitting over a selected range of frequencies, and finally providing the natural frequencies, mod shapes, and modal damping factors.

5.5. Fixture and Test-Specimen: A fixture holds the steel bar of rectangular cross section firmly at one end so that the specimen can be considered as a cantilever beam. The cross-sectional dimensions of the cantilever beam used are 19mm x 2 mm and length 300 mm. The fixture holds the cantilever bar firmly at one end and the fixture is assembled to a table. A small accelerometer is attached at the middle of the cantilever beam using an adhesive like loctite.

6. NATURAL FREQUENCY FROM EXPERIMENTATION

Figure No. 2: Experimental Setup

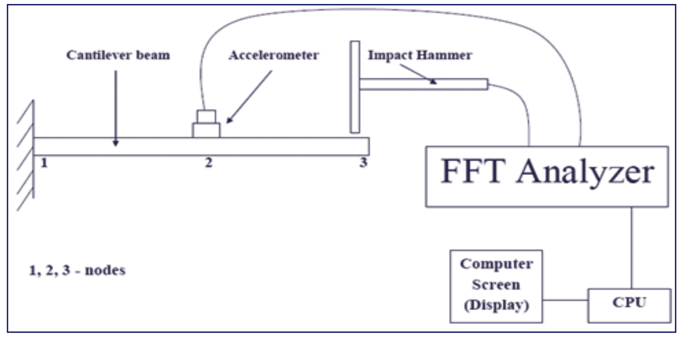


Figure No. 3: Actual Photograph of Experimental Setup

Prepare the cantilever beam. Measure the length on the fixture that holds the steel cantilever beam and leave a margin of that length on the steel cantilever beam. Divide the remaining length of the steel cantilever beam into 2 parts and mark node numbers at each division from 1 to 3. Let node 3 be the free end and node 1 the fixed one. Fix the accelerometer to the steel cantilever beam at the middle of the beam. Make sure that the face of the cantilever beam with markings and node numbers up, fix the beam into the slot on the fixture so that a cantilever is formed. Connect all the wires and cables properly. Make connections of the vibration analyzer, computer or laptop, accelerometer and the impact hammer. Switch on the power supply. Open the software of vibration analysis and experimental modal analysis installed on the computer/laptop. Provide necessary inputs and make necessary settings in the software. Ensure that there is proper supply and communication between the devices connected. Now we shall provide impacts by the impact hammer on the nodes marked on the cantilever beam one by one. Impacts will be given on nodes 2, 3; node 1 is fixed. Accelerometer is connected at node 2. Signals from the impact hammer and the accelerometer will be received by the vibration analyzer for each impact provided one by one and will be compared and analyzed by the software. Curve known as Frequency Response Function (FRF) will be generated by the software that is used to find the natural frequencies of the cantilever beam. Observe the curve and read frequencies that correspond to peaks of the FRF.

7. OBSERVATION GRAPHS FROM FFT ANALYZER

Figure No. 4: The first mode undamped natural frequency of cantilever beam

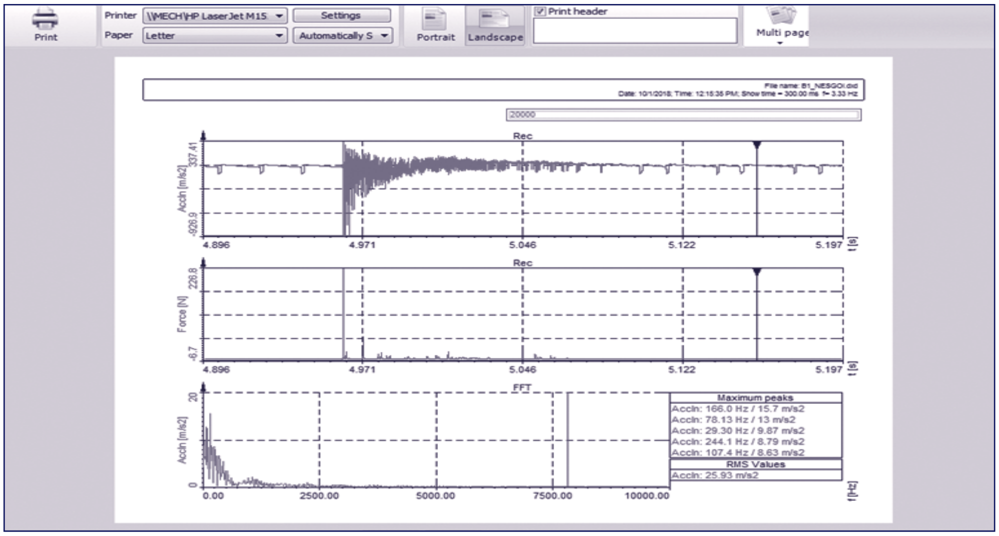


Figure No. 5: The second mode undamped natural frequency of cantilever beam

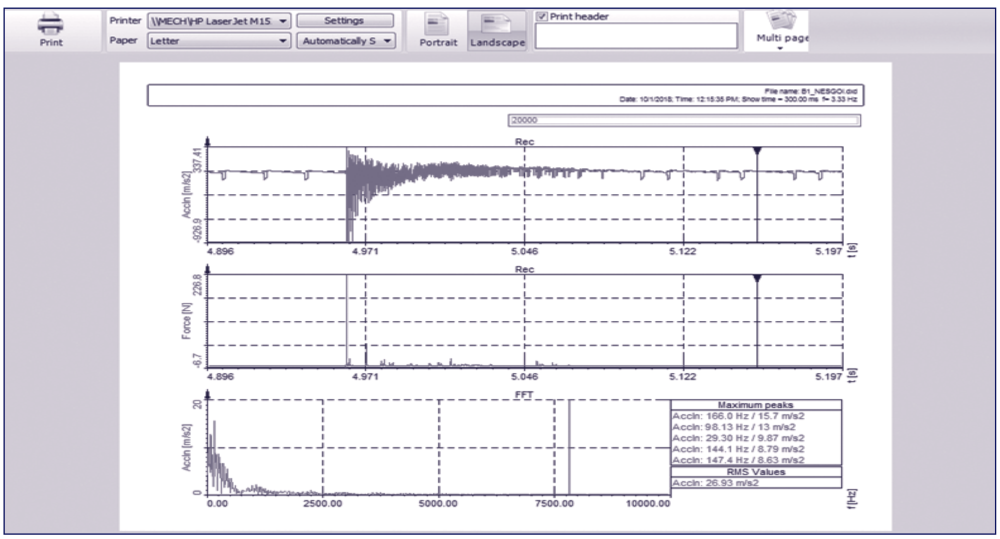
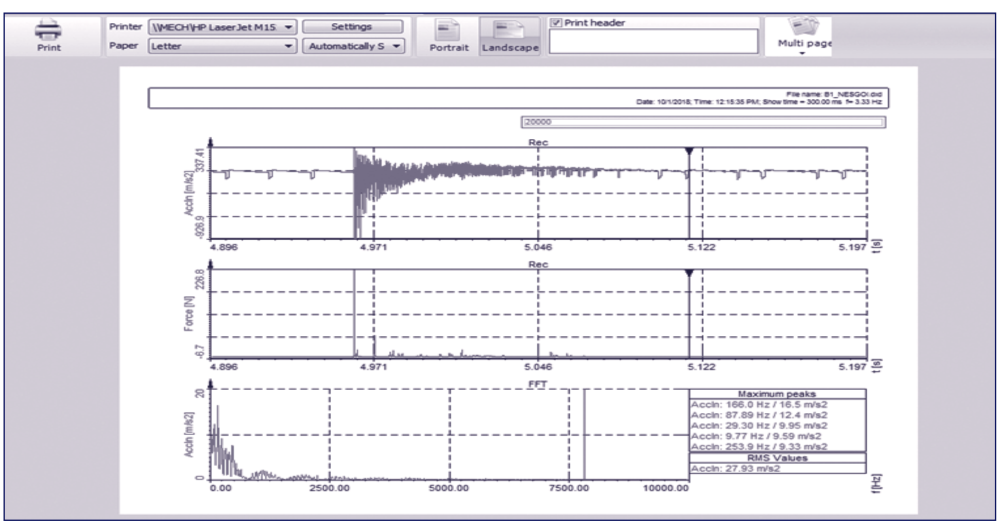


Figure No. 6: The third mode undamped natural frequency of cantilever beam



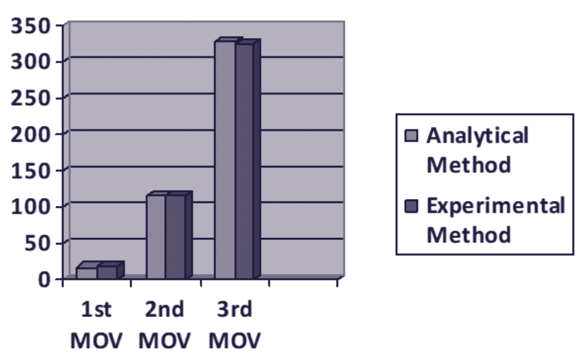
Observed the FRF (Mode Identification Function). The frequencies that correspond to the peaks of amplitude are noted. These corresponds to the natural frequencies of the cantilever beam. In fact, they correspond to the natural frequencies of the principal modes of the cantilever beam that were excited, or participated in vibration of the cantilever beam due to impacts. Compared these values with those obtained from formulae. Comparative analysis is done on the difference in values obtained from these two methods experimental and the analytical.

8. ANALYTICAL AND EXPERIMENTAL RESULTS

Table 1: Analytical and experimental results

Mode of vibration	Natural frequency by Analytical Calculation (Hz) (Analytical Method)	Natural frequency by FFT Analyzer (Hz) (Experimental Method)
1st	18.62 Hz	19.77Hz
2nd	116.74Hz	116.0 Hz
3rd	326.9Hz	324.0Hz

Figure No. 7: Analytical and experimental results



9. CONCLUSIONS

Vibration analysis is one of the most important processes in the field of mechanical engineering. Cantilever beam vibration study is very important. It helps to determine the durability concerns. This information from this work can be is used to reduce the discomfort and excessive stresses in different applications in which beams are essential components. Beams are exposed to different dynamic loads in different applications. When a cantilever beam is Subjected to a dynamic load, the beam will vibrate at its natural frequencies. The natural frequency of vibration of cantilever beam is analyzed by both analytical and experimental methods and both the results were compared. From result table it can be concluded that Natural frequency obtained by FFT Analyzer i.e., by experimental method is matching with natural frequency by analytical calculation. The dimensions of the cantilever beam are length L = 0.3 m, breadth b = 0.019 m and thickness t = 0.002 m. Natural frequency obtained by analytical calculation for 1st mode, 2nd mode and 3rd mode of vibrations are 18.62 Hz, 116.74Hz and 326.9Hz respectively. The natural frequency obtained by experimental method by using FFT for 1st mode, 2nd mode and 3rd mode of vibrations are 19.77Hz, 116.0 Hz and 324.0Hz.

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